**Weekly Focus:** composite solids
**Weekly Skill:** find dimensions, applications

## LESSON 49: Composite Solids

**Lesson Summary**: For the warm-up, students will solve a problem about the earth and sun. In Activity 1, they will review finding the area of 2-D figures. In Activity 2, students will examine examples of finding the volume and the surface area of composite solids. In Activity 3, they will do word problems. The first application activity is about the volume of discarded cigarette butts. The second (optional) application activity is about comparing a single-flush with a dual-flush toilet. Estimated time for the lesson is 2 hours.

**Materials Needed for Lesson 49:**

- Meter sticks for Application Activity 1
- Video (length 7:00) that is a good summary review of the volume and surface area of solids taught in the two previous lessons.
- Video (length 4:17) on volume of a composite cylinder, sphere, and cone.
- Video (3:42) on surface area of composite solids. (The videos are required for teachers and recommended for students.)
- Notes A (attached) are from: http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf (pages 30 – 31)
- Notes B (attached)
- 1 Worksheet (49.1) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 110 – 111)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 158 – 161)
- The 1st application activity (attached) is from: http://robertkaplinsky.com/work/cigarette-butts/
- The 2nd optional application activity (attached) is from: http://robertkaplinsky.com/work/toilet/

**Objectives**: Students will be able to:

- Solve the earth and sun word problem
- Calculate the area of composite 2-D figures (review from lesson 45)
- Calculate the volume and surface area of composite solids
- Solve one or two real-life application problems involving ratios, volume, and/or algebra

**ACES Skills Addressed:** N, CT, LS
**CCRS Mathematical Practices Addressed:** Building Solution Pathways, Make Sense of Problems and Persevere in Solving Them, Model with Math
**Levels of Knowing Math Addressed:** Intuitive, Pictorial, Abstract, and Application

**Notes:**
**You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.**

**For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.**

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

| Lesson 49 Warm-up: Solve the Earth and Sun Problem | Time: 10 Minutes |
|---|---|

Write on the board: The radius of the earth is about 6,370 km. The radius of the sun is about 695,000 km.

Basic Questions:

- What is the diameter of the earth?
  - The radius x 2 = 12,740 km
- What is the diameter of the sun?
  - The radius of the sun x 2 = 1,390,000 km
- What is the ratio of the earth's radius to the sun's radius?
  - $\frac{6370}{695{,}000} \approx \frac{1}{109}$ The sun's radius is over 100 times the earth's radius.

Extension Questions:

- What is the surface area of the earth?
  - $SA = 4\pi r^2 = 4\,(3.14)(6{,}370^2) \approx 510{,}000{,}000$
- Write the surface area in scientific notation
  - $5.1 \times 10^8$
- Note: It would take about 1 million Earths to fill the Sun if it were a hollow ball.

| Lesson 49 Activity 1: Area of Composite 2-D Figures | Time: 15 Minutes |
|---|---|

1. The objective of this activity is for students to review finding the area of 2-dimensional shapes before they try to find the area of more difficult 3-dimensional solids.
2. Note: Students did an activity like this one in Lesson 45, but it is worth repeating to review.
3. Give students **Worksheet 49.1**. Do #1 on the board as an example.
4. Ask students what shapes they "see" in this figure. They may say a rectangle and half a circle.
5. Draw it on the board with a broken line to divide the half circle from the rectangle. Point out that the width of the rectangle is also the diameter of the circle. Solve the area of the rectangle ($A = 6 \times 10 = 60\ m^2$) and add to the area of ½ a circle ($A= \frac{1}{2}(3.14)(3^2)= 14.13m^2$ to get a total area of $60 + 14.13 = 70.13\ m^2$.
6. Students can do the rest of the problems independently. If necessary, have volunteers do 1-2 problems on the board.

| Lesson 49 Activity 2: Volume and Surface Area Examples of Composite Solids | Time: 15 Minutes |
|---|---|

1. Copy **Notes A, pages 30 – 31,** to give to students as an example of finding the volume of a composite solid.
2. A **composite solid** is one that is made of two or more solids. The one in this example is composed of a cylinder with a hemisphere (half of a sphere) at each end.
3. Go through the example on the board. In the notes, they wait until the end to mention that the two hemispheres make one sphere, but you may want to point that out right away to make the calculations faster. Even better, ask the students what they see at the two ends.
4. Also copy **Notes B** to do as an example to solve for the surface area of a composite solid.
5. Work through the example together.

| Lesson 49 Activity 3: Composite Solids Computation and Word Problems | Time: 45 Minutes |
|---|---|

1. Do the problems in the **student book** pages 110-111 together.
2. Students can do the problems in the **workbook** pages 158-161 independently.
3. Circulate to help and solve some of the problems on the board as needed.

| Lesson 49 Application Activity 1: How Big a Problem Are Littered Cigarette Butts | Time: 20-30 Minutes |
|---|---|

1. This is an interesting activity that includes graphs, volume measurement and other computations. It also increases environmental awareness.
2. Become familiar with the activity before presenting it to students.
3. Students can measure the volume of the classroom with meter sticks and figure out how many classrooms would be filled with those cigarette butts instead of the cafeteria or stadium options mentioned in the activity.

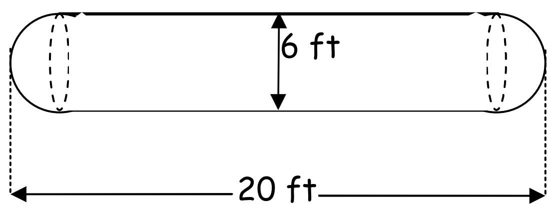
| Lesson 49 Application Activity 2: Which Toilet Uses Less Water? | Time: 20-30 Minutes |
|---|---|

1. This is a fun (and funny) culminating activity. It includes ratios, volume, and even systems of equations from algebra.
2. Become familiar with the activity before presenting it to the students.

3. Give students time to discuss and struggle to get to a reasonable answer. Provide more or less support to them depending on their needs, but do help them with the idea of systems of equations, which you may need to review first.
4. Start the activity by showing the photo of the dual flush toilet and asking the students if they are familiar with it. Also ask if they know its purpose.

**Notes A: Volume of Composite Solids Example**

A propane gas tank consists of a cylinder with a hemisphere at each end. Find the volume of the tank if the overall length is 20 feet and the diameter of the cylinder is 6 feet.




Solution:

We are told this tank consists of a cylinder (one its side) with a hemisphere at each end. A hemisphere is half of a sphere. To find the volume, we need to find the volume of the cylinder and the volumes of each hemisphere and then adding them together.

Let's start with the hemisphere sections. The diameter of the circular base of the cylinder is indicated to be 6 feet. This would also be the diameter of the hemispheres at each end of the cylinder. We need the radius of the sphere to find its volume. Once we calculate the volume of the whole sphere, we multiply it by ½ to find the volume of the hemisphere (half of a sphere). We calculate the radius to be $r = \frac{1}{2}d = \frac{1}{2}(6) = 3$ feet. Now we can calculate the volume of the two hemispheres at the ends of the tank.

Left Hemisphere:

The volume of a whole sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(3^3\right) = 36\pi$ cubic feet. We now multiply this volume by ½ to find the volume of the hemisphere to get $\frac{1}{2}V = \frac{1}{2}\left(36\pi\right) = 18\pi$ cubic feet.

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

**r = the radius of the sphere**
**π = the number that is approximated by 3.141593**

Right Hemisphere:

The volume of a whole sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(3^3\right) = 36\pi$ cubic feet. We now multiply this volume by $\frac{1}{2}$ to find the volume of the hemisphere to get $\frac{1}{2}V = \frac{1}{2}\left(36\pi\right) = 18\pi$ cubic feet.

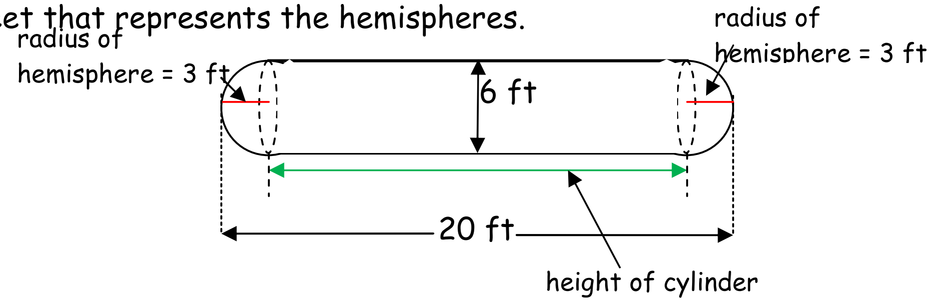
You might notice that we went through exactly the same process with exactly the same number for each hemisphere. We could have shortened this process by realizing that putting together the two hemispheres on each end, which were of the same diameter, would create a whole sphere. We could just have calculated the volume of this whole sphere.

**Volume of a Cylinder**

$V = Ah$

**A = the area of the base of the cylinder**
**h = the height of the cylinder**

Now we need to calculate the volume of the cylinder. We need the radius of the cylinder and the height of the cylinder to find its volume. The radius of the cylinder is the same as the radius of the hemispheres at each end. Thus the radius of the cylinder is 3 feet. It may look like the height of the cylinder is 20 feet. It turns out that this is not the case. The 20 feet includes the hemispheres at each end. We need to subtract the part of the 20 feet that represents the hemispheres.

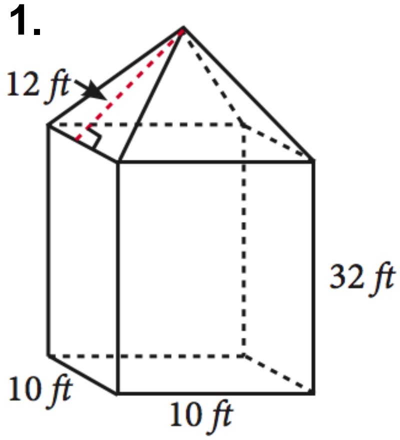



Looking at the figure above, we see that the distance from the end of the left hemisphere to the left end of the cylinder is the radius of the hemisphere. The radius of the hemisphere is 3 feet. Similarly, the distance from the right end of the cylinder to the end of the right cylinder is also the radius of the hemisphere. The radius of the hemisphere is 3 feet. If we now subtract these two distances from the overall length of the tanks, we will have the

**Notes B: Surface Area of a Composite Solid Example**

Find the surface area of the figure below:

**1.**




Identify the parts of the solid on the surface.

Find the area of the base.

$$A_{Base} = 10(10) = 100$$

Find the lateral area of the prism.

$$LA_{Prism} = P_{base}h$$
$$LA_{Prism} = 4(10)(32) = 1280$$

Find the lateral area of the pyramid.

$$LA_{Pyramid} = \frac{1}{2}P_{base}h$$
$$LA_{Pyramid} = \frac{1}{2}(40)(12) = 240$$

Find the sum of all three parts.

$$S.A. = 1280 + 240 + 100$$

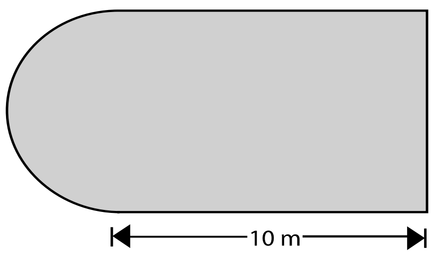
The surface area of the composite solid is 1620 square feet.

## Worksheet 49.1 Area of Composite 2-D Figures

### Area - Compound Shapes

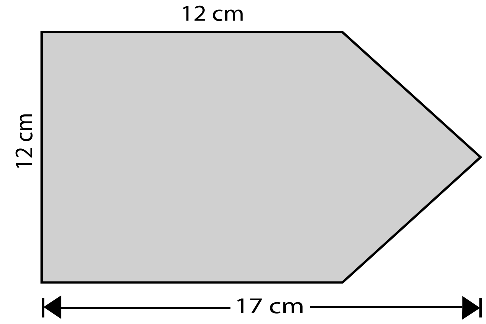
Find the area of each figure. Round tha answer to 2 decimal places if necessary.

1)



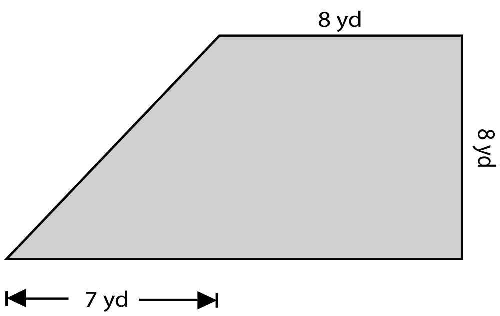

Area = ____________

2)



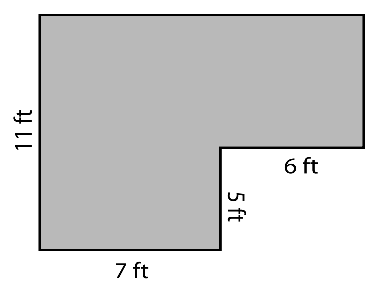

Area = ____________

3)



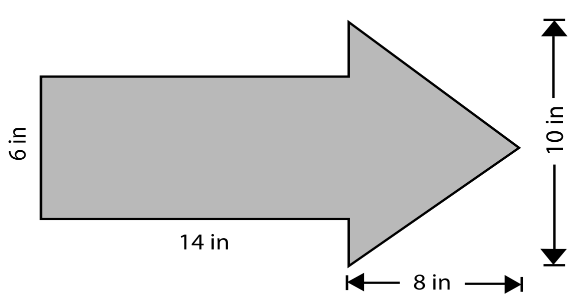

Area = ____________

4)



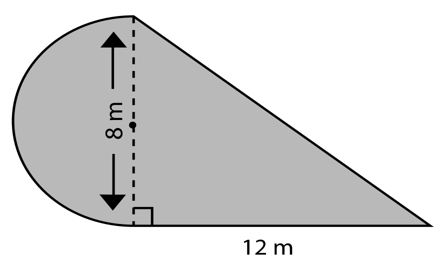

Area = ____________

5)




Area = ____________

6)



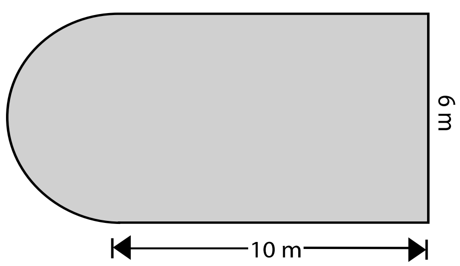

Area = ____________

## Worksheet 49.1 Answers

**Answer Key**

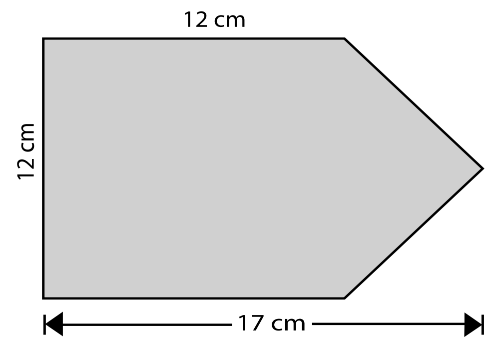
Find the area of each figure. Round tha answer to 2 decimal places if necessary.

1)



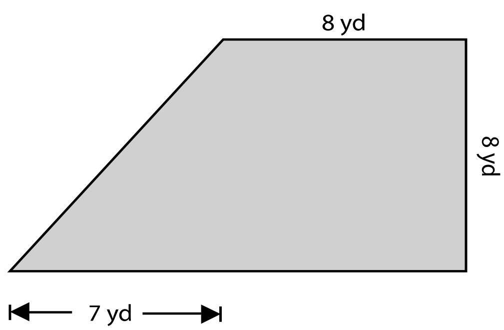

Area = **$74.13\ m^2$**

2)



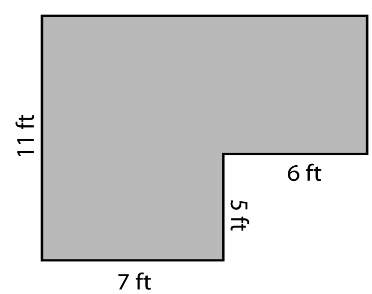

Area = **$174\ cm^2$**

3)



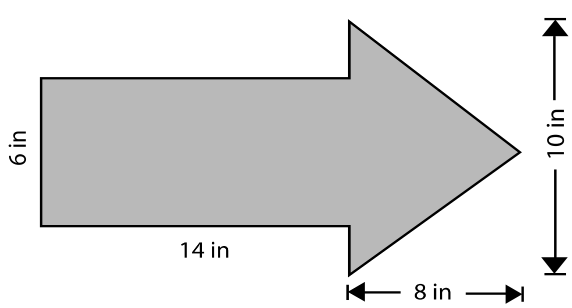

Area = **$92\ yd^2$**

4)



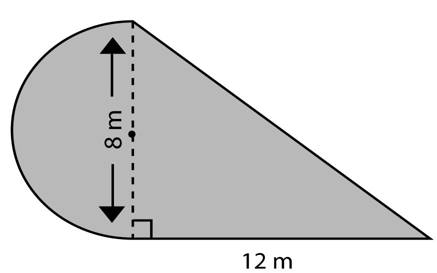

Area = **$113\ ft^2$**

5)




Area = **$124\ in^2$**

6)




Area = **$73.12\ m^2$**

**Lesson 49 Application Activity 1: How Big a Problem Are Littered Cigarette Butts?**

robertkaplinsky.com http://robertkaplinsky.com/work/cigarette-butts/

## How Big of a Problem are Littered Cigarette Butts?

**The Situation**

### The Challenge(s)

- How big of a building would all the cigarette butts littered every day in the United States fill? A classroom? An auditorium? A stadium?
- How long would it take to fill a ____________?
- How long would a line of cigarette butts littered each day measure?

### Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What information would be useful in figuring this out?
- What factors may affect your answer's accuracy?
- What is a guess for ______________ that is too low?

### Consider This

- It has been said that "There are three kinds of lies: lies, damned lies, and statistics." As such, it is worth discussing the general accuracy of statistics with students.
- Students may begin by finding the number of smokers (43.5 million as of 2010) and on average how many cigarettes they smoke each day (14.1 as of 2004).
- Using that information and that only 90% of cigarette butts are littered (10% are properly disposed), you get 552,015,000 cigarette butts littered each day.
- With that number of littered cigarette butts you can multiply it by an estimate for the volume of a cigarette butt (1.758 cubic centimeters based on King Size and a cylinder with a ~8 mm diameter and ~35 mm height) using its dimensions.
- This gives a volume of ~970 cubic meters. Students can then determine the volume of a classroom and cafeteria. The volumes of an auditorium and stadium are given.

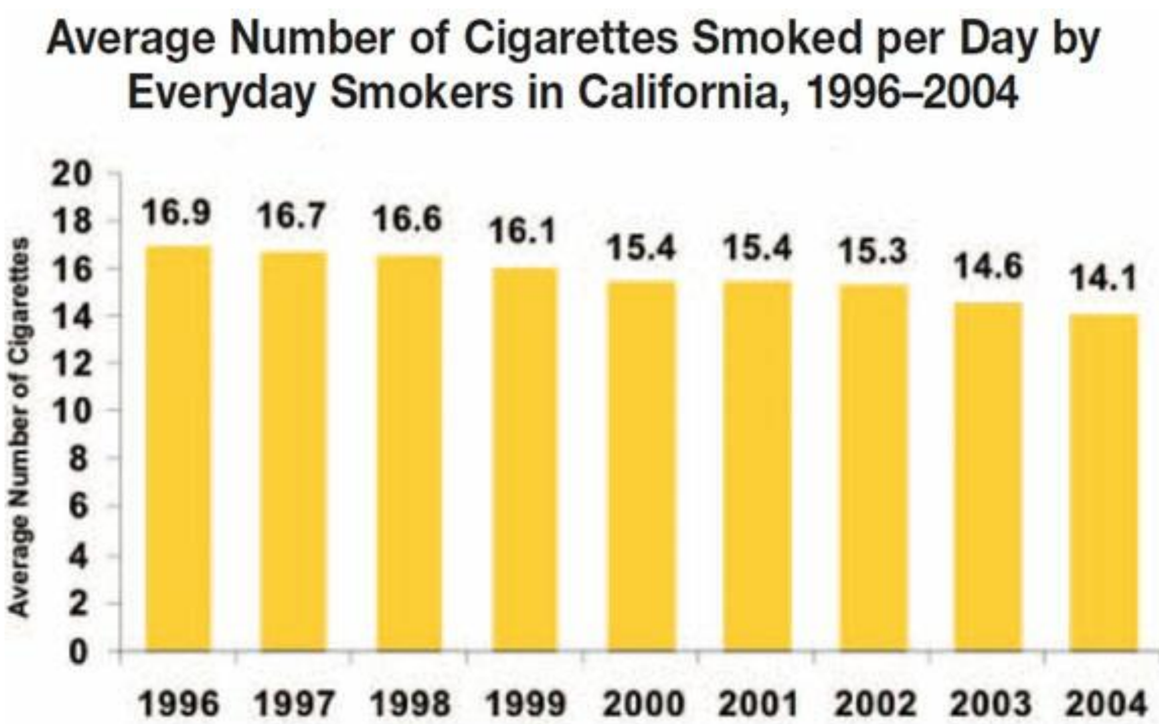
**What You'll Need**

- Number of smokers in the United States:

## Number of Smokers and Number of Smokers Who Have Quit

- In 2010, 43.5 million adults (19.3%) in the United States were current smokers—21.5% of men and 17.3% of women.

- Average number of cigarettes smoked each day:




Source: Behavioral Risk Factor Surveillance System (BRFSS) and California Adult Tobacco Survey (CATS) 1996–2004. The data is weighted to the 1990 California population.
Prepared by California Department of Health Services, Tobacco Control Section, May 2005.

- Percentage of cigarettes littered:

**CIGARETTE LITTERING MISCONCEPTIONS**

Only 10% of cigarette butts are properly deposited in ash receptacles-the least likely item to be placed in a receptacle.[1]

- Volume of an auditorium:

The Great Auditorium, with volume of 90,000 cubic meters, seats 3,693 in the lower auditorium, 3,515 in the balcony, 2,518 in the gallery and 300 to 500 on the dais. Government leaders make their speeches; and the representatives do much of their business. It can simultaneously seat 10,000 representatives. The ceiling is decorated with a galaxy of lights, with a large red star is at the centre of the ceiling, and a pattern of a water waves nearby represents the people. Its facilities equipped with audio-visual and other systems adaptable to a variety of meeting types and sizes. A simultaneous interpretation system is also provided with a language booth.

- Volume of a stadium:

**Structure**

- The stadium contains 2,618 toilets, more than any other venue in the world.[25]
- The stadium has a circumference of 1 km (0.62 mi).[26]
- The bowl volume is listed at 1,139,100 $m^3$, somewhat smaller than the Millennium Stadium in Cardiff, but with a greater seating capacity.

**Lesson 49 Application Activity 2: Which Toilet Uses Less Water?**

robertkaplinsky.com http://robertkaplinsky.com/work/toilet/

## Which Toilet Uses Less Water?

## The Situation

One type of toilet comes in two models: dual flush or single flush. The dual flush toilet has two buttons: one flushes "solids" with more water and "liquids" with less water. The single flush toilet always flushes with the same amount of water.

## The Challenge(s)

- When would both toilets use the same amount of water?

## Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What information do we need to know?
- When would the dual flush toilet obviously use less water?
- When would the single flush toilet obviously use less water?

- How could the two toilets use about the same amount of water?
- What assumptions are we making?

## Consider This

I have gone back and forth about whether to include this as a lesson. On the one hand, it is a real world situation where I had to choose the toilet that made the most economic and environmental sense. On the other hand it will make teenage boys and girls talk about going potty in class.

The information that you need to know (and what may not be easily readable on the image) is the amount of water each toilet uses. The toilets use the unit gpf or gallons (of water used) per flush. The dual flush toilet uses 1.6 gpf for "solids" and 1.0 gpf for "liquids" while the single flush always uses 1.28 gpf.

From my experiences, people approach this problem from one of two ways:

- Some attempt this problem as a series of guess and check ratios. Specifically, they know that the single flush toilet always uses 1.28 gpf. So, they try to find out what ratio of "liquids" to "solids" on the dual flush toilet also gives an average of 1.28 gpf. They might start with 1:1 and realize that averages 1.3 gpf. Then they might try 2:1 and realize that averages 1.2 gpf. From there they narrow down the ratio until they eventually get to 8:7 which has a ratio of exactly 1.28 gpf.

- Others attempt this problem using one of these two systems of equations (which is my ultimate goal for this lesson):
    - 1.0*x + 1.6*y = total water usage AND 1.28*x +1.28*y = total water usage
        - Where x is the number of “liquid” flushes and y is the number of “solid” flushes
        - Set then equal to each other and solve
    - x*1.0 + y*1.6 = 1.28 AND x + y = 1
        - Where x is the percentage of “liquid” flushes and y is the percentage of “solid” flushes
        - The first equation describes the percentage of flushes for “liquids” and “solids” that would average out to 1.28 gpf
        - The second equation has those two percentages adding up to 1 (or 100%)

If students get stuck, consider sharing these three scenarios:

- You use the dual flush toilet 100% of the time for “liquids”.  So, it uses 1.0 gpf which is less water than the 1.28 gpf single flush toilet.
- You use the dual flush toilet 100% of the time for “solids”.  So, it uses 1.6 gpf which is more water than the 1.28 gpf single flush toilet.
- You use the dual flush toilet 50% of the time for “liquids” and 50% of the time for “solids”.  So, it averages 1.3 gpf which is more than the 1.28 gpf single flush toilet.

Using this type of reasoning, students should be able to reach the conclusion that the toilets’ water usage would be about the same when the dual flush toilet is used slightly less than 50% of the time for “solids” and slightly more than 50% of the time for “liquids”.

One last note: for a historical perspective, most toilets used to use 3.4 gpf.  Today toilets now use 1.6 gpf or even 1.28 gpf.  So, both of these toilets would save water compared to older models.

I have also left the prices in the pictures in case you want to also find out when one toilet would pay for itself from the water savings.


Dual Flush
Descarga Doble
1.0
gallon flush for liquids
1.6
gallon flush for solids
American Standard
American Standard
Item
Model
Clean™ Cadet® 3
Overall dimensions: 15 in W x 31 in H x 29-3/4 in D
Rough-in dimensions: 12 in
Trapway size: 2 in
• High-efficiency, dual flush toilet—1.6 gal. or 1.0 gal. flush
• Stays cleaner longer with EverClean® surface & PowerWash™ flush
• Features No Tools™ installation
• ADA approved chair height
Elongated
18.5 in
46.99 cm
Chair Height
16.5 in
41.91 cm
10 Year
Limited Warranty
$199
Everyday Low Price
CLEAN CADET 3 DUAL FLUSH CH EL H
40 W 1


American Standard
Item | Articulo: 88575
Model | Modelo: 2514.101.020
Clean™ Cadet® 3
Overall dimensions: 15-3/4 in W x 30-3/4 in H x 30-1/4 in D
Rough-in dimensions: 12 in
Trapway size: 2-1/16 in
• Smooth-sided toilet design
• Stays cleaner longer with EverClean® surface & PowerWash™ flush
• Features No Tools™ installation
• ADA approved chair height
Elongated
18.5 in
46.99 cm
Chair Height
16.5 in
41.91 cm
10 Year
Limited Warranty
$239
Everyday Low Price
ASD CLEAN CADET3 EL CH 1.28GPF
40 W 5