Weekly Focus: composite figures
Weekly Skill: measurement

LESSON 45: Composite Plane Figures

Lesson Summary: For the warm up, students will solve a problem about the costs of pie crusts. In Activity 1, students will calculate the area and perimeter of composite figures. In Activity 2, they will solve problems independently. The Application Activity is about estimating the amount of money found in a drug bust. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 45:

- Video (length 6:00) on calculating the area of composite figures. The video is required for teachers and optional for students.
- 1 Worksheet (45.1) with answers (attached)
- Application Activity (http://robertkaplinsky.com/work/drug-money/)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 102 – 103)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 142 – 145)

Objectives: Students will be able to:

- Solve the review word problem about the cost of pie crusts
- Calculate the area and perimeter of composite figures
- Solve word problems about composite figures
- Estimate how much money was found in a real-life drug bust

ACES Skills Addressed: N, CT, LS, EC
CCRS Mathematical Practices Addressed: Model with Math, Reason Abstractly and Quantitatively
Levels of Knowing Math Addressed: Intuitive, Pictorial, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 45 Warm-up: Solve the pie crust problem	**Time: 10 Minutes**

Write on the board: Nora is baking pies for a holiday party and wants to buy frozen pie crusts. They are sold in packs of 2 for $5.50 or 3 for $7.80.

Basic Questions:

- Which is the better deal?
  - The 3 pack = $2.60 per crust
- She also has a coupon for $1 off. What is her final cost per crust?
  - $7.80- $1 = $6.80/3 = $2.27 per crust

Extension Questions:

- Nora's pie pans have an area of 50 in^2. Should she buy small crusts with 6-inch diameters or medium crusts with 8-inch diameters?
  - Medium, because a diameter of 8 inches = radius of 4 inches.
  - $A= 3.14\ r^2= 3.14\ (4)^2 = 3.14\ (16) =$ about 50 in^2

Lesson 45 Activity 1: Calculate the area and perimeter of composite figures	**Time: 25 Minutes**

1. Composite figures are those made up of 2 or more shapes.
2. Their area or perimeter can be calculated by dividing the composite shape into smaller ones.
3. Copy **Worksheet 45.1**. Do each one as an example of finding the area.
4. Also solve for perimeter if there is enough time.
5. Do **pages 102-103 in the student book** together. Have volunteers do #2 and #5 on the board to show how they partitioned the shapes and did their calculations.

Lesson 45 Activity 2: Solve composite figure measurement problems	**Time: 45 Minutes**

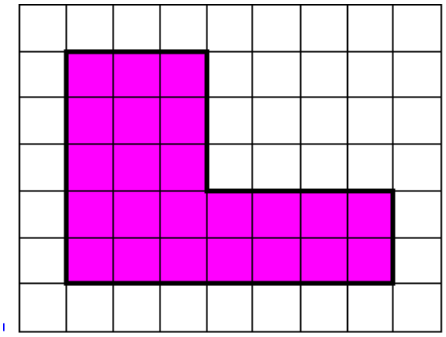
1. Students can work independently on **pages 142-145 in the workbook**.
2. Circulate to help.
3. Have volunteers do some of the more challenging problems on the board.

Lesson 45 Application: How Much Money is That?	**Time: 25 Minutes**

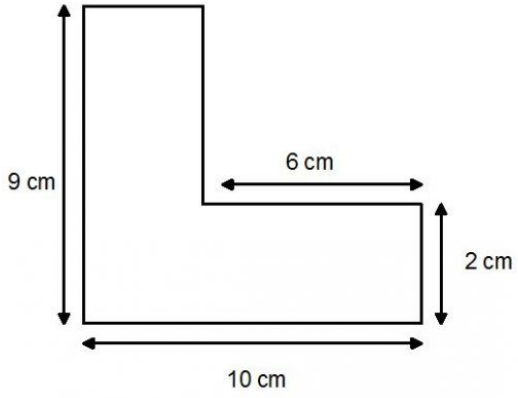
1. This activity is not about composite figures, but about measuring volume and doing computation with money. It is related to a real-life drug bust.
2. Notes:
    a. Become familiar with the activity before presenting it to the students. Go to the website (link is given above) if possible for access to the news clip about the drug bust. Also, you can reference sample student work on the website.
    b. Give students time to discuss before helping them solve the problem.

Worksheet 45.1

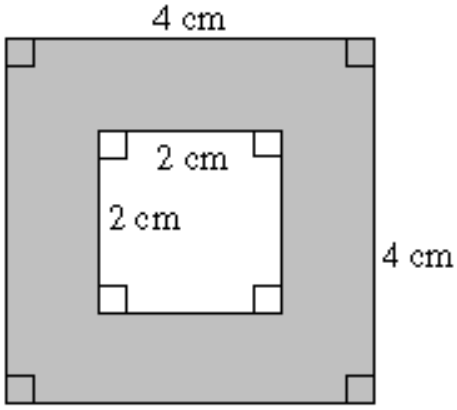
1. Find the area of the shaded region of the figure below. Each small square is 1 cm x 1cm.



2. A piece of cardboard is cut in an L-shape as shown below. Find the total area of the cardboard.



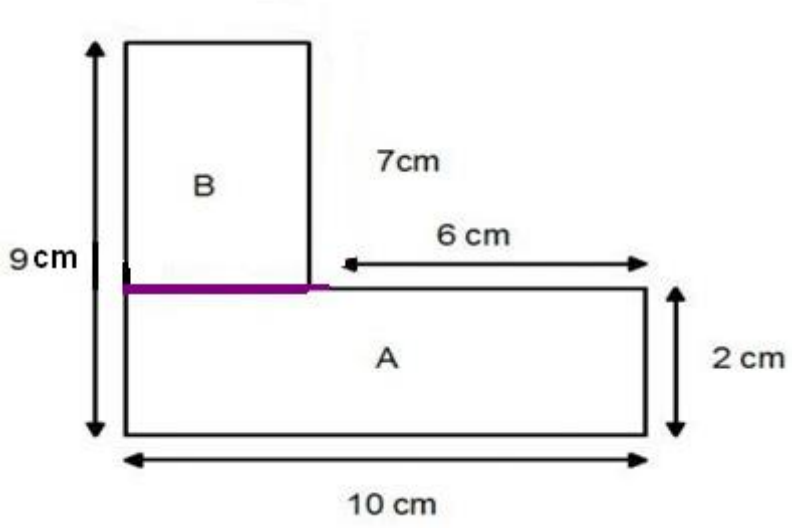
3. Find the area of the shaded region of the figure below.



Worksheet 45.1 Answers

Answer 1: The shaded region has 23 squares. Each small square is 1 cm x 1cm. Hence the shaded area is 23 square cm.

Answer 2: This is an L-shaped figure and can be split into two rectangular parts A and B.



Area A = 10 cm x 2 cm = 20 square cm

Area B = 4 cm x 7 cm = 28 square cm

Total Area = Area A + Area B = 48 square cm

Answer 3: Area of shaded region = Area of the larger square – Area of the small white square

= 4 cm x 4 cm – 2 cm x 2 cm = 12 square cm

Answer 4: The complex shape can be split into a square A (10x10) and a triangle B with a base of10 and a height of 10 unit.

Area of square A = 100 square unit

Area of triangle B = ½ x base x height = ½ x 10x 10= 50 square unit

Total area of the figure = 150 square unit

Lesson 45 Application

robertkaplinsky.com http://robertkaplinsky.com/work/drug-money/

How Much Money IS That?!

The Situation

The Challenge(s)

- How much money is that?

Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What is a guess that is too low?
- What is a guess that is too high?
- What is your best guess?
- What options do you have for counting this money and what are the advantages and disadvantages to each method?
- How can we measure the volume of $100 bills in the pile?
- How important is it that we have an accurate answer?

Consider This

How do you count money when there is so much it is "stuffed to the ceiling?" It is a problem that I wish I had. These pictures give a much clearer picture of how much money there actually is.

Picture 1

Picture 2

Picture 3

A few notes about the money:

- My assumption is that all of the United States bills are $100 bills.

- I am ignoring the colorful money in front of the United States bills as they do not appear to be US currency.
- Looking at Picture 1 (with help from Picture 3) it looks like the main stack of bills go back 11 rows with a slightly less high 12th row in the front.
- Looking at Picture 1 (with help from Picture 2) it looks like the pile has 34 columns of bills.

The most important part of this lesson will be the mathematical modeling conversation regarding how to go about counting the money. So, after getting students' guesses, be sure to have a long talk around "What options do you have for counting this money and what are the advantages and disadvantages to each method?" Here are my thoughts on what students might say:

- You could count each bill, one at a time.
  - Advantage: very accurate if you don't lose track of the running total
  - Disadvantage: very time consuming
- You could count the bills using an electric bill counter (as clearly you would be able to afford one!)
  - Advantage: potentially the most accurate if you don't lose track of the running total
  - Disadvantage: also very time consuming as you can only put stacks of up to 130 bills at a time
- You could hire multiple people to count them for you.
  - Advantage: potentially faster than the previous two methods but still time consuming
  - Disadvantage: they could steal your money and you have to pay them
- You could estimate how much money is in the pile based on its weight
  - Advantage: should take less time than counting the individual bills
  - Disadvantage: limited accuracy as the scale may not be precise and the bills' weight may change from handling
- You could estimate how much money is in the pile based on its volume
  - Advantage: by far the fastest method
  - Disadvantage: limited accuracy

Ultimately you should guide students to see that while the first four options are reasonable, the only one we can complete based on the information we have is option 5 where we estimate how much money there is based on its volume. This is an important opportunity for students to build conceptual understanding of volume instead of skipping straight to procedures. Give students the images and let them strategize about "How can we measure the volume of $100 bills in the pile?" There are quite a few options including the ones below so help guide students to see the connections between them:

- Find the amount of money in one stack of bills and multiply it by the number of stacks.
- Find the amount of money in one layer of bills and multiply it by the number of layers.
- Find the amount of money in one row of bills and multiply it by the number of rows.

- Find the amount of money in one column of bills and multiply it by the number of columns.
- Find area in square units of one layer of bills and divide it by the area of one bill, then multiply it by the number of layers.

It is worth noting the the first four options are the easiest and do not use the traditional volume procedure of measuring the length, width, and height in linear units and multiplying them all together. Regarding the last option above, it is one of the few I could think of that begins with such an approach.

In terms of coming up with the estimate, here is how I approached it:

- Main pile of money:
  - There are 10 rows and 34 columns of money in the main pile so there are 340 stacks of money.
  - To estimate height, the largest stack of paper whose count I know are reams of paper. I estimate that highest stacks of money are as tall as 15 reams of paper.
  - Reams of copy paper hold 500 sheets while reams of cardstock often hold 250 sheets. The money itself is used and likely crinkled so I estimate that it is between the thickness of copy paper and cardstock but closer to copy paper. I estimate that a stack of used bills that are the same height as a ream of paper is 400 bills.
  - Multiplying this all together: 340 stacks * 15 reams tall * 400 bills per ream * $100 per bill gives a total for the main stack of $204 million
- Shorter single row of bills in the front
  - I can see from Picture 2 that it is also 34 columns across giving us 34 stacks of bills.
  - The stacks on the left half appear to be about two reams of paper shorter than the main pile of money behind it.
  - The stacks on the right half appear to be about two reams of paper short than the left half.
  - Averaging the two I get that they are about 12 reams of paper tall.
  - Multiplying this all together: 34 stacks * 12 reams tall average * 400 bills per ream * $100 per bill gives a total for the main shorter single row of bills of $16,320,000.
- Thus, the grand total I have is $220,320,000.

Once students have their estimates in, play this clip for them. Note that there is such a large margin of error in their estimates that the news anchor says “200 million” while the text says “205 million.

Make sure to have students discuss why they think their answer was different from what the news anchor stated. For example, I think my answer was a little high because of my estimation of the number of bills in each stack. If I had estimated that there were 372 bills per ream, I would have been right on the money.

According to this article from the Los Angeles Times, they state more specifically that there was $205.6 million in cash. Other interesting details they include are:

- “the cash seized was mostly in U.S. $100 bills and weighed at least 4,500 pounds.”
- “experts were still analyzing the $205.6 million in cash to check for counterfeits but that the bills appeared to be legitimate.”
- “Officials with the attorney general’s organized crime unit used a moving truck, guarded by a 25 patrol-car caravan, to take the money to its headquarters.”